

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

8623401314

FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

May/June 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10 \,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

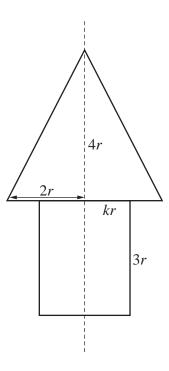
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A particle P of mass m is attached to one end of a light inextensible string of length a. The other end

i ina, in terms	of a and g , the time the	hat <i>P</i> takes to make one complete revolution.	
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A particle <i>Q</i> o magnitude <i>mkv</i>	f mass $m \log f$ falls from N, where $v \log^{-1} \log g$	om rest under gravity. The motion of Q is resisted the speed of Q at time t s and k is a positive constant	by a forc
	f mass $m \log f$ falls from N, where $v \log^{-1} \log r$ is sion for v in terms of		by a forc
			by a force

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C	oo wiiii iile i	upwaru veru	cai unougn	O. The speed	ı or Q when i	t is at A is $2\sqrt{a}$	ig.	
Given that T	T_A and T_B are	the tensions	in the string	g at A and B	respectively, f	find the ratio T_{i}	$T_A:T_B.$	[6
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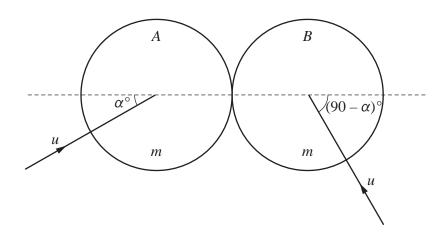


A uniform solid circular cone, of vertical height 4r and radius 2r, is attached to a uniform solid cylinder, of height 3r and radius kr, where k is a constant less than 2. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram). The cone and the cylinder are made of the same material.

(a)	Show that the distance of the centre of mass of the combined solid from the vertex of the cone is $(99k^2 + 96)r$
	$\frac{(99k^2 + 96)r}{18k^2 + 32}. ag{4}$

The point C is on the circumference of the base of the cone. When the combined solid is freely suspended from C and hanging in equilibrium, the diameter through C makes an angle α with the downward vertical, where $\tan \alpha = \frac{1}{8}$.

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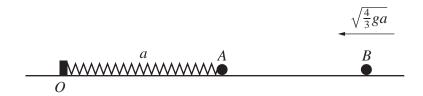
Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion makes an angle of α° with the line of centres, and B's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e.

Immediately after the collision, *B* moves in a direction at right angles to the line of centres.

(a)	Show that $\tan \alpha = \frac{1+e}{1-e}$.	[4]

)	plan	article P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal ne and moves freely under gravity. The direction of motion of P makes an angle α above the izontal when P first reaches three-quarters of its greatest height.
	(a)	Show that $\tan \alpha = \frac{1}{2} \tan \theta$. [6]

	Given that $\tan \theta = \frac{4}{3}$, find the horizontal distance travelled by <i>P</i> when it first reaches throf its greatest height. Give your answer in terms of <i>u</i> and <i>g</i> .	
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One end of a light spring of natural length a and modulus of elasticity 4mg is attached to a fixed point O. The other end of the spring is attached to a particle A of mass km, where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a. A second particle B, of mass m, is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).

The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

(a)	Find the value of k .	[6]
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At the point C the horizontal surface becomes rough, with coefficient of friction μ between the combined particle and the surface. The deceleration of the combined particle at C is $\frac{9}{20}g$.

(b)	Find the value of μ .	[4]
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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.			

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